Series RLC Circuit Analysis

Series RLC circuits consist of a resistance, a capacitance and an inductance connected in series across an alternating supply.

Thus far we have seen that the three basic passive components of: Resistance, Inductance, and Capacitance have very different phase relationships to each other when connected to a sinusoidal alternating supply.

In a pure ohmic resistor the voltage waveforms are “in-phase” with the current. In a pure inductance the voltage waveform “leads” the current by 90°, giving us the expression of: ELI. In a pure capacitance the voltage waveform “lags” the current by 90°, giving us the expression of: ICE.
This Phase Difference, $\Phi$ depends upon the reactive value of the components being used and hopefully by now we know that reactance, $(X)$ is zero if the circuit element is resistive, positive if the circuit element is inductive and negative if it is capacitive thus giving their resulting impedances as:

**Element Impedance**

<table>
<thead>
<tr>
<th>Circuit Element</th>
<th>Resistance, $(R)$</th>
<th>Reactance, $(X)$</th>
<th>Impedance, $(Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>$0$</td>
<td>$Z_R = R$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= R \angle 0^0$</td>
</tr>
<tr>
<td>Inductor</td>
<td>$0$</td>
<td>$\omega L$</td>
<td>$Z_L = j\omega L$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= \omega L \angle 90^0$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$0$</td>
<td>$-\frac{1}{\omega C}$</td>
<td>$Z_C = \frac{1}{j\omega C}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= \frac{1}{\omega C} \angle -90^0$</td>
</tr>
</tbody>
</table>

Instead of analysing each passive element separately, we can combine all three together into a series RLC circuit. The analysis of a **series RLC circuit** is the same as that for the dual series $R_L$ and $R_C$ circuits we looked at previously, except this time we need to take into account the magnitudes of both $X_L$ and $X_C$ to find the overall circuit reactance. Series RLC circuits are classed as second-order circuits because they contain two energy storage elements, an inductance $L$ and a capacitance $C$. Consider the RLC circuit below.

**Series RLC Circuit**
The series RLC circuit above has a single loop with the instantaneous current flowing through the loop being the same for each circuit element. Since the inductive and capacitive reactance's $X_L$ and $X_C$ are a function of the supply frequency, the sinusoidal response of a series RLC circuit will therefore vary with frequency, $f$. Then the individual voltage drops across each circuit element of $R$, $L$ and $C$ element will be “out-of-phase” with each other as defined by:

$$i(t) = I_m \sin(\omega t)$$

The instantaneous voltage across a pure resistor, $V_R$ is “in-phase” with current

The instantaneous voltage across a pure inductor, $V_L$ “leads” the current by 90°

The instantaneous voltage across a pure capacitor, $V_C$ “lags” the current by 90°

Therefore, $V_L$ and $V_C$ are 180° “out-of-phase” and in opposition to each other.

For the series RLC circuit above, this can be shown as:
The amplitude of the source voltage across all three components in a series RLC circuit is made up of the three individual component voltages, \( V_R \), \( V_L \) and \( V_C \) with the current common to all three components. The vector diagrams will therefore have the current vector as their reference with the three voltage vectors being plotted with respect to this reference as shown below.

**Individual Voltage Vectors**
This means then that we can not simply add together $V_R$, $V_L$ and $V_C$ to find the supply voltage, $V_S$ across all three components as all three voltage vectors point in different directions with regards to the current vector. Therefore we will have to find the supply voltage, $V_S$ as the Phasor Sum of the three component voltages combined together vectorially.

Kirchhoff’s voltage law (KVL) for both loop and nodal circuits states that around any closed loop the sum of voltage drops around the loop equals the sum of the EMF’s. Then applying this law to the these three voltages will give us the amplitude of the source voltage, $V_S$ as.

**Instantaneous Voltages for a Series RLC Circuit**

$$\text{KVL: } V_S - V_R - V_L - V_C = 0$$

$$V_S - IR - L \frac{di}{dt} - \frac{Q}{C} = 0$$

$$\therefore V_S = IR + L \frac{di}{dt} + \frac{Q}{C}$$

The phasor diagram for a series RLC circuit is produced by combining together the three individual phasors above and adding these voltages vectorially. Since the current flowing through the circuit is common to all three circuit elements we can use this as the reference vector with the three voltage vectors drawn relative to this at their corresponding angles.

The resulting vector $V_S$ is obtained by adding together two of the vectors, $V_L$ and $V_C$ and then adding this sum to the remaining vector $V_R$. The resulting angle obtained between $V_S$ and $i$ will be the circuits phase angle as shown below.

**Phasor Diagram for a Series RLC Circuit**
We can see from the phasor diagram on the right hand side above that the voltage vectors produce a rectangular triangle, comprising of hypotenuse $V_S$, horizontal axis $V_R$ and vertical axis $V_L - V_C$. Hopefully you will notice then, that this forms our old favourite the **Voltage Triangle** and we can therefore use Pythagoras’s theorem on this voltage triangle to mathematically obtain the value of $V_S$ as shown.

**Voltage Triangle for a Series RLC Circuit**

\[
V_S^2 = V_R^2 + (V_L - V_C)^2
\]

\[
V_S = \sqrt{V_R^2 + (V_L - V_C)^2}
\]

Please note that when using the above equation, the final reactive voltage must always be positive in value, that is the smallest voltage must always be taken away from the largest voltage we can not have a negative voltage added to $V_R$ so it is correct to have $V_L - V_C$ or $V_C - V_L$. The smallest value from the largest otherwise the calculation of $V_S$ will be incorrect.
We know from above that the current has the same amplitude and phase in all the components of a series RLC circuit. Then the voltage across each component can also be described mathematically according to the current flowing through, and the voltage across each element as.

\[ V_R = iR \sin(\omega t + 0^0) = iR \]

\[ V_L = iX_L \sin(\omega t + 90^0) = i.j\omega L \]

\[ V_C = iX_C \sin(\omega t - 90^0) = i.\frac{1}{j\omega C} \]

By substituting these values into the Pythagoras equation above for the voltage triangle will give us:

\[ V_R = iR \quad V_L = iX_L \quad V_C = iX_C \]

\[ V_S = \sqrt{(i.R)^2 + (i.X_L - i.X_C)^2} \]

\[ V_S = i.\sqrt{R^2 + (X_L - X_C)^2} \]

\[ \therefore V_S = i \times Z \quad \text{where: } Z = \sqrt{R^2 + (X_L - X_C)^2} \]

So we can see that the amplitude of the source voltage is proportional to the amplitude of the current flowing through the circuit. This proportionality constant is called the **Impedance** of the circuit which ultimately depends upon the resistance and the inductive and capacitive reactance's.
Then in the series RLC circuit above, it can be seen that the opposition to current flow is made up of three components, $X_L$, $X_C$ and $R$ with the reactance, $X_T$ of any series RLC circuit being defined as: $X_T = X_L - X_C$ or $X_T = X_C - X_L$ whichever is greater. Thus the total impedance of the circuit being thought of as the voltage source required to drive a current through it.

**The Impedance of a Series RLC Circuit**

As the three vector voltages are out-of-phase with each other, $X_L$, $X_C$ and $R$ must also be “out-of-phase" with each other with the relationship between $R$, $X_L$ and $X_C$ being the vector sum of these three components. This will give us the RLC circuits overall impedance, $Z$. These circuit impedance’s can be drawn and represented by an **Impedance Triangle** as shown below.

**The Impedance Triangle for a Series RLC Circuit**

$$Z^2 = R^2 + (X_L - X_C)^2$$

The impedance $Z$ of a series RLC circuit depends upon the angular frequency, $\omega$ as do $X_L$ and $X_C$. If the capacitive reactance is greater than the inductive reactance, $X_C > X_L$ then the overall circuit reactance is capacitive giving a leading phase angle.

Likewise, if the inductive reactance is greater than the capacitive reactance, $X_L > X_C$ then the overall circuit reactance is inductive giving the series circuit a lagging phase angle. If the two reactance’s are the same and $X_L = X_C$ then the angular frequency at
which this occurs is called the resonant frequency and produces the effect of resonance which we will look at in more detail in another tutorial.

Then the magnitude of the current depends upon the frequency applied to the series RLC circuit. When impedance, $Z$ is at its maximum, the current is a minimum and likewise, when $Z$ is at its minimum, the current is at maximum. So the above equation for impedance can be re-written as:

$$\text{Impedance, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phase angle, $\theta$ between the source voltage, $V_S$ and the current, $i$ is the same as for the angle between $Z$ and $R$ in the impedance triangle. This phase angle may be positive or negative in value depending on whether the source voltage leads or lags the circuit current and can be calculated mathematically from the ohmic values of the impedance triangle as:

$$\cos \phi = \frac{R}{Z}, \quad \sin \phi = \frac{X_L - X_C}{Z}, \quad \tan \phi = \frac{X_L - X_C}{R}$$

**Series RLC Circuit Example No1**

A series RLC circuit containing a resistance of 12Ω, an inductance of 0.15H and a capacitor of 100uF are connected in series across a 100V, 50Hz supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram.
Inductive Reactance, $X_L$.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13 \Omega$$

Capacitive Reactance, $X_C$.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83 \Omega$$

Circuit Impedance, $Z$.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4 \Omega$$
Circuits Current, I.

\[ I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14 \text{Amps} \]

Voltages across the Series RLC Circuit, \( V_R, V_L, V_C \).

\[ V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts} \]
\[ V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts} \]
\[ V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts} \]

Circuits Power factor and Phase Angle, \( \theta \).

\[ \cos \phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619 \]

\[ \therefore \cos^{-1} 0.619 = 51.8^\circ \text{ lagging} \]

Phasor Diagram.
Since the phase angle $\theta$ is calculated as a positive value of $51.8^\circ$ the overall reactance of the circuit must be inductive. As we have taken the current vector as our reference vector in a series RLC circuit, then the current “lags” the source voltage by $51.8^\circ$ so we can say that the phase angle is lagging as confirmed by our mnemonic expression “ELI”.

**Series RLC Circuit Summary**

In a **series RLC circuit** containing a resistor, an inductor and a capacitor the source voltage $V_S$ is the phasor sum made up of three components, $V_R$, $V_L$ and $V_C$ with the current common to all three. Since the current is common to all three components it is used as the horizontal reference when constructing a voltage triangle.

The impedance of the circuit is the total opposition to the flow of current. For a series RLC circuit, and impedance triangle can be drawn by dividing each side of the voltage triangle by its current, $I$. The voltage drop across the resistive element is equal to $I^*R$, the voltage across the two reactive elements is $I^*X = I^*X_L - I^*X_C$ while the source voltage is equal to $I^*Z$. The angle between $V_S$ and $I$ will be the phase angle, $\theta$.

When working with a series RLC circuit containing multiple resistances, capacitance’s or inductance’s either pure or impure, they can be all added together to form a single component. For example all resistances are added together, $R_T = (R_1 + R_2 + R_3)...$etc.
or all the inductance's \( L_T = (L_1 + L_2 + L_3) \)...etc this way a circuit containing many elements can be easily reduced to a single impedance.

In the next tutorial about Parallel RLC Circuits we will look at the voltage-current relationship of the three components connected together this time in a parallel circuit configuration when a steady state sinusoidal AC waveform is applied along with the corresponding phasor diagram representation. We will also introduce the concept of Admittance for the first time.

143 Comments

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K sreenathreddy

May I know theory for determine RLC values for given RLC network

Posted on August 28th 2018 | 1:37 pm

a anik

how can i determine the r (the d.c resistance of the inducter due to the coil ?

Posted on August 20th 2018 | 5:40 pm
Wayne Storr

Measure the coil with an ohmmeter or digital tester

Posted on August 20th 2018 | 5:59 pm

Maria

Draw the impedance frequency graph for two different values of resistance of the circuit and interpret it

Posted on August 09th 2018 | 1:45 am

Vyankatesh

Plz electrical projects

Posted on August 07th 2018 | 1:57 am

Venkatesan

Hi sir
Two resister and two inductance present in circuit what value in circuit

Posted on August 06th 2018 | 7:23 am
Fabian Hilary Osome

the solution to the problem has really backed up my knowledge on RLC circuit. I Thank you so much.

Posted on July 28th 2018 | 12:37 pm

Umesh Kumar

Good

Posted on July 24th 2018 | 7:10 am

Natasha ng'andu

I have understood the RCL circuit. Thanks

Posted on July 21st 2018 | 3:58 pm

Nampita rajab

Teach me more

Posted on July 17th 2018 | 10:02 am
What does the capacitor voltage lagging the applied voltage by 40 degrees imply? Need your assistance on whether the RLC circuit is more inductive or more capacitive.

Posted on July 09th 2018 | 3:12 pm